Tailoring two-photon spontaneous emission using atomically thin plasmonic nanostructures

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March 2nd, 2020

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CAPES

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

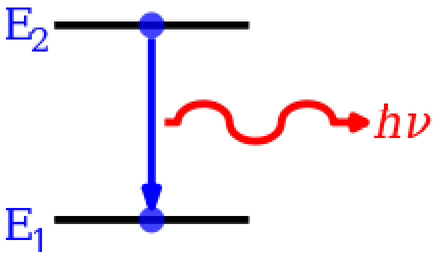
Los Alamos NATIONAL LABORATORY EST. 1943

Center for Nonlinear Studies

A brief introduction about spontaneous emission

Spontaneous emission (SE)

- An **excited atom**, even when isolated, **decays** to its fundamental state.

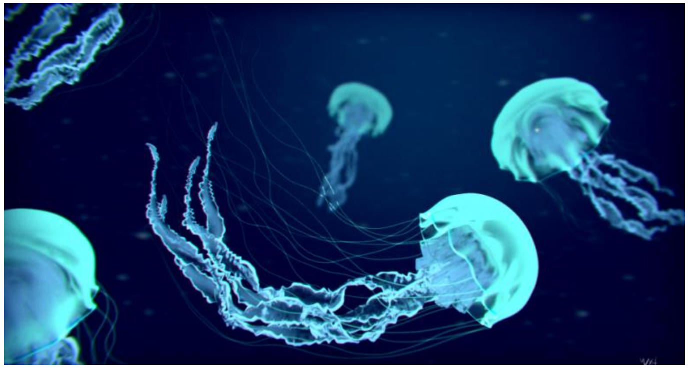


- Phenomenon induced by **quantum vacuum fluctuations**.

- Quantum electrodynamics (QED): excited atom + zero photons is not a stationary state of the atom-field system.

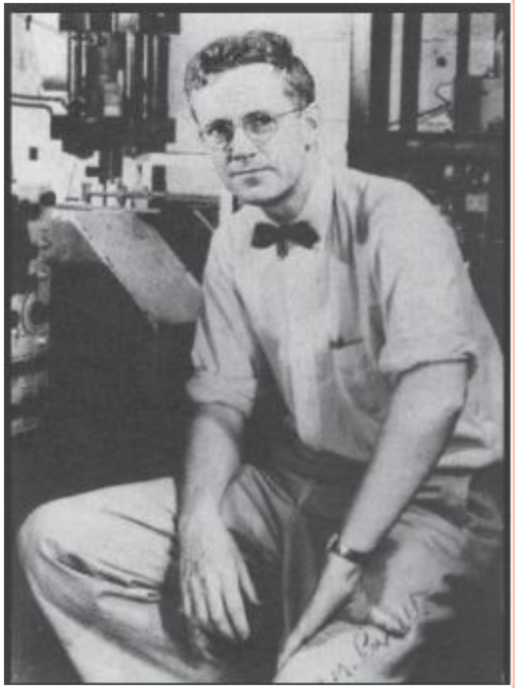
SE

- Most of the light we see is from SE.



PURCELL EFFECT

- E.M. **Purcell (1946)**: Bodies in the vicinities of an emitter change its SE rate.



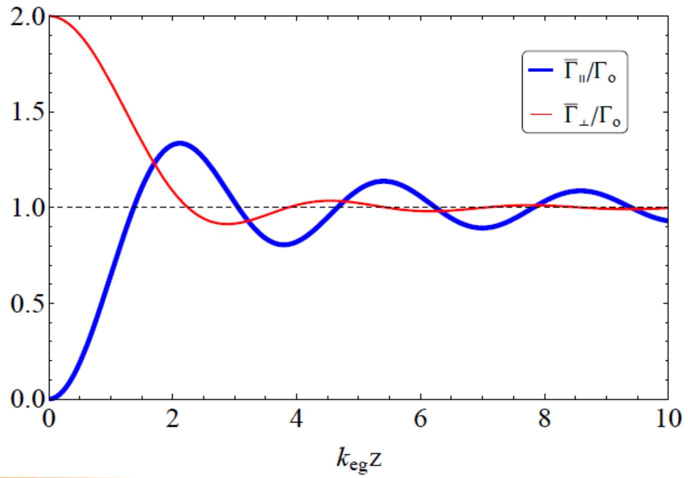
- Reason: The presence of the bodies affects the **boundary conditions** (BC) on the electromagnetic field.

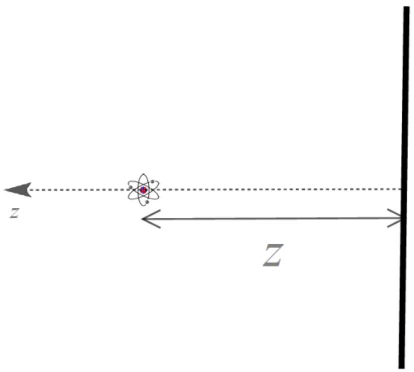
$$\Gamma(\mathbf{R}) = \frac{\pi}{\epsilon_o \hbar} \sum_{\mathbf{k}p} \omega_k |\mathbf{d}_{eg} \cdot \mathbf{A}_{\mathbf{k}p}(\mathbf{R})|^2 \delta(\omega_k - \omega_{eg}).$$

- It can be shown that the SE rate is proportional to the local density of states (**LDOS**) of the electromagnetic field.

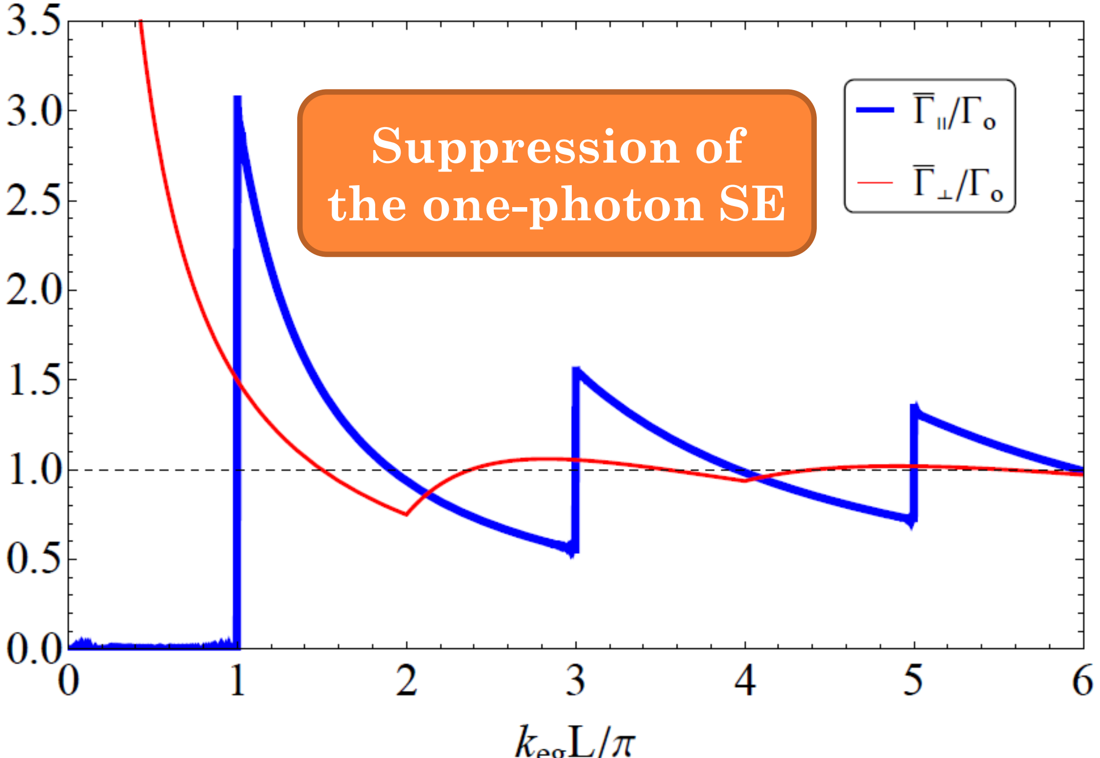
L. Novotny and **B. Hecht**, *Principles of nano-optics*. Cambridge university press, 2012.

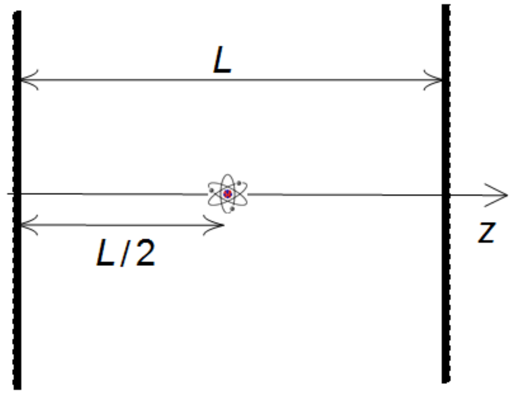
Purcell effect on the one-photon SE





Morawitz, Phys. Rev (**1969**)





Hulet et al., PRL (**1985**)

Two-photon spontaneous emission (TPSE)

- **Second order process** in perturbation theory (**Göppert-Mayer, 1931**).
- Relevant process when the one-photon SE is forbidden, for instance, due to **selection rules**.
- Ex: **2s – 1s** transition in **H(Breit, Teller, 1940**). $\tau \approx 1/7s$
- **Broadband** spectrum of emission.
- Explains the emission spectrum of planetary nebulae.

L. Spitzer and J. L. Greenstein, The Astrophysical Journal, vol. **114**, p. **407 (1951)**.

Purcell effect on the TPSE

- **Not widely discussed** in the literature.

- The progress in **near-field optics, plasmonics,** and **materials science** in general has improved our **control** over **radiation-matter interactions**.

- In some situations the TPSE can even dominate conventionally fast transitions!

N. Rivera *et al*, "*Making two-photon processes dominate one-photon processes using mid-ir phonon polaritons*", PNAS, p. **201713538 (2017**)

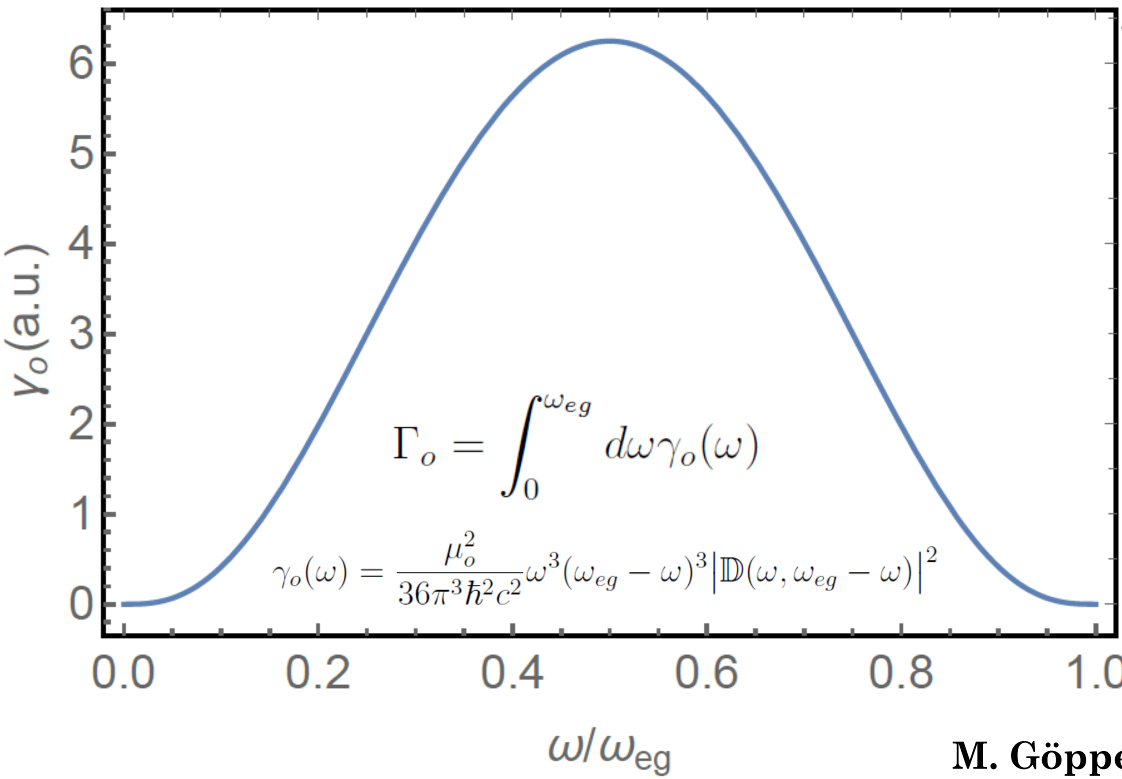
- TPSE is a rich phenomenon, with very much to be explored yet.

TPSE RATE: FIELD MODES APPROACH

- **Second order Fermi's golden rule** gives

$$\Gamma(\mathbf{R}) = \frac{\pi}{4\epsilon_o^2\hbar^2} \sum_{\mathbf{k}p,\mathbf{k}'p'} \omega_k\omega_{k'} \left|\mathbf{A}_{\mathbf{k}p}(\mathbf{R}) \cdot \mathbb{D}(\omega_k, \omega_{k'}) \cdot \mathbf{A}_{\mathbf{k}'p'}(\mathbf{R})\right|^2 \delta(\omega_k + \omega_{k'} - \omega_{eg}),$$

$$\mathbb{D}(\omega_k, \omega_{k'}) := \lim_{\eta\to 0^+} \sum_m \left[\frac{\mathbf{d}_{em}\mathbf{d}_{mg}}{\omega_{em} - \omega_k + i\eta} + \frac{\mathbf{d}_{mg}\mathbf{d}_{em}}{\omega_{em} - \omega_{k'} + i\eta}\right].$$

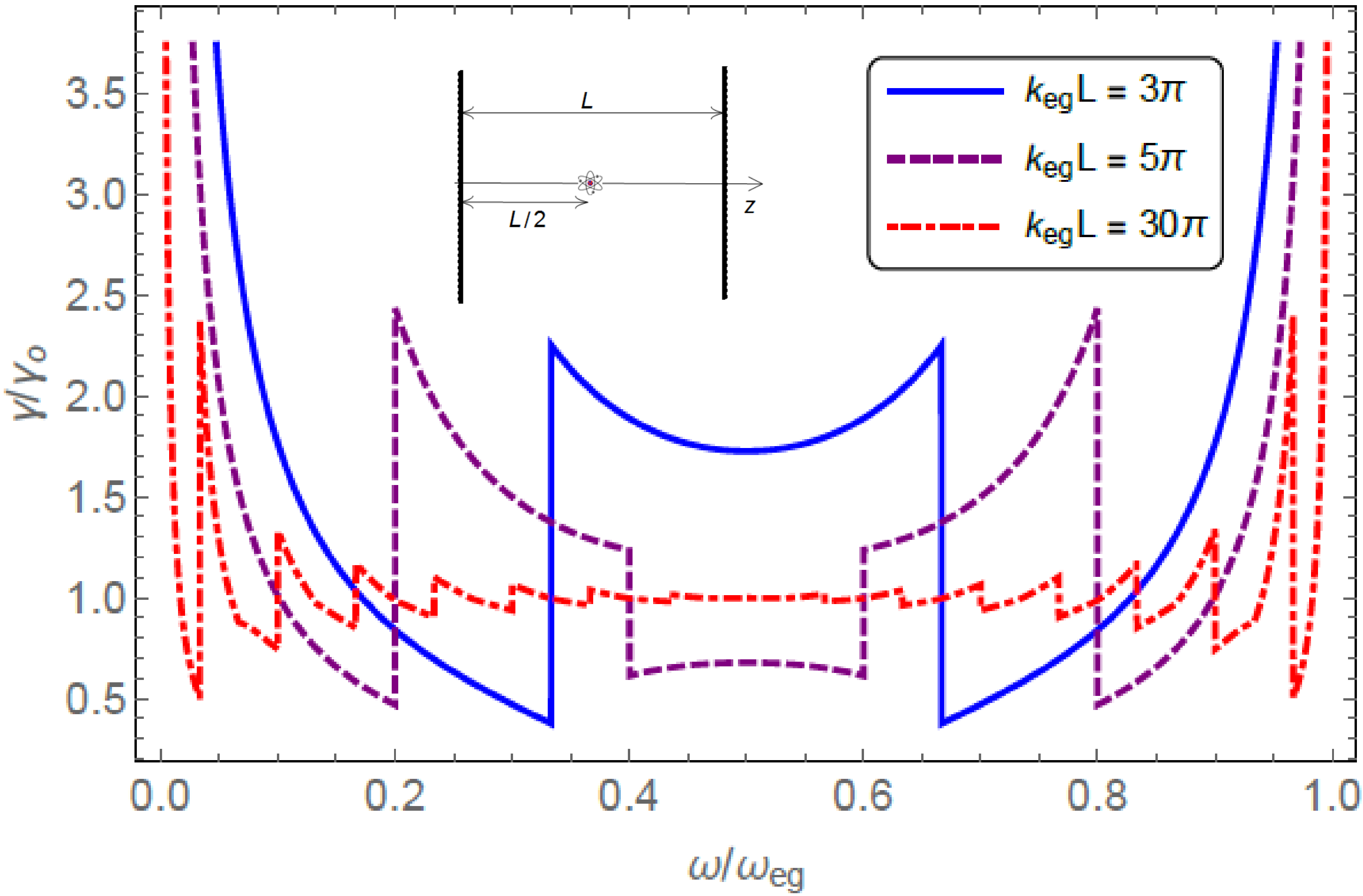


Y.M. *et al*, PRA, **100**, 023818 **(2019)**

Free space TPSE spectral distribution

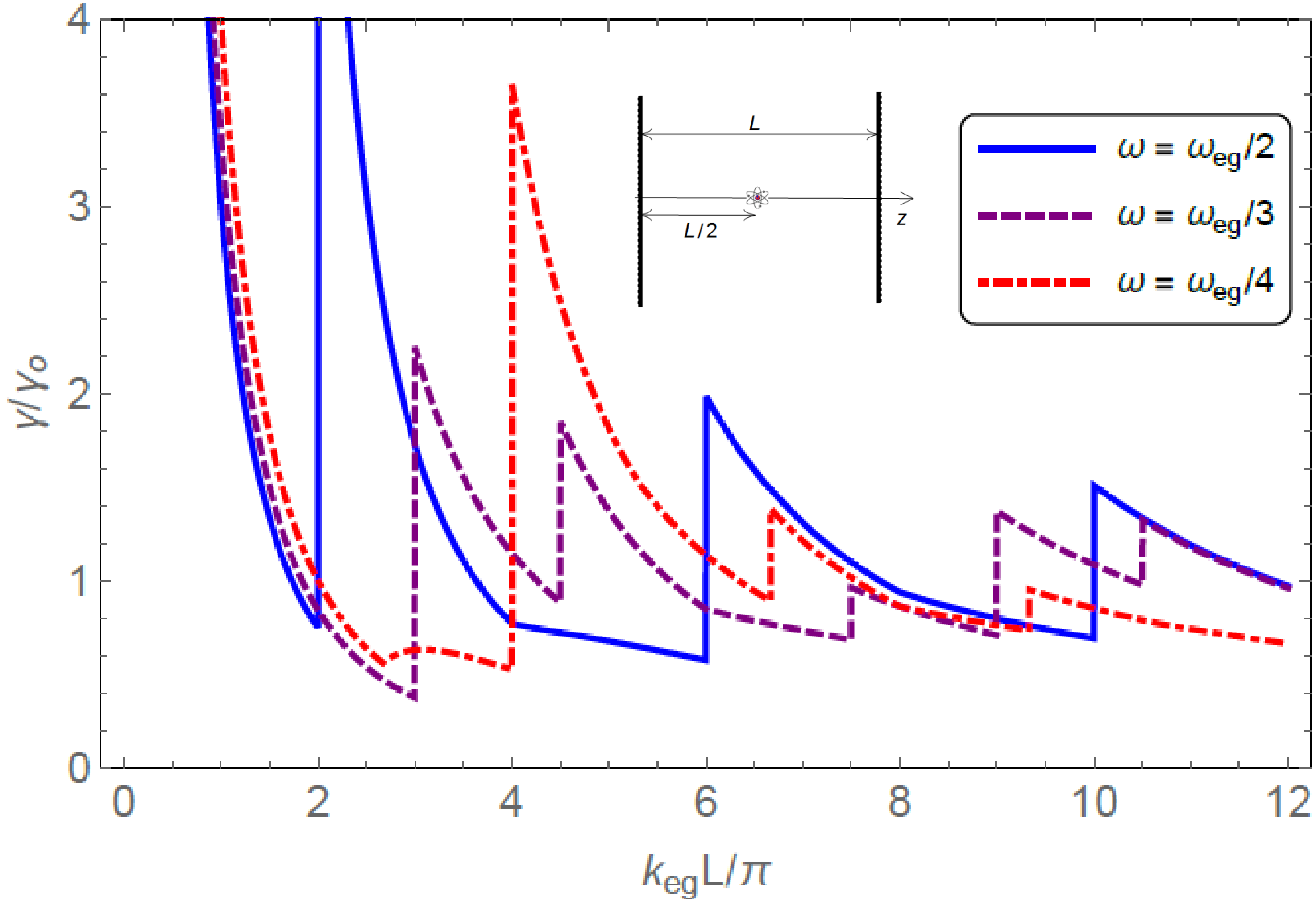
M. Göppert-Mayer, Annalen der Physik **(1931)**

An emitter between two perfect mirrors (s → s)



Abrupt changes in the spectral density due to discontinuities in the LDOS.

An emitter between two perfect mirrors (s → s)



The TPSE is not suppressed, in contrast to what happens to the one-photon SE in this situation.

Relation between TPSE and one-photon SE

- It is possible to show that

$$\Gamma(\mathbf{R}_e) = \int_0^{\omega_t} d\omega \gamma_0(\omega) \sum_{a,b} t_{ab}(\omega) P_a(\mathbf{R}_e, \omega) P_b(\mathbf{R}_e, \omega_t - \omega)$$

$$t_{ab}(\omega) = |\mathbb{D}_{ab}(\omega, \omega_t - \omega)|^2 / |\mathbb{D}(\omega, \omega_t - \omega)|^2$$

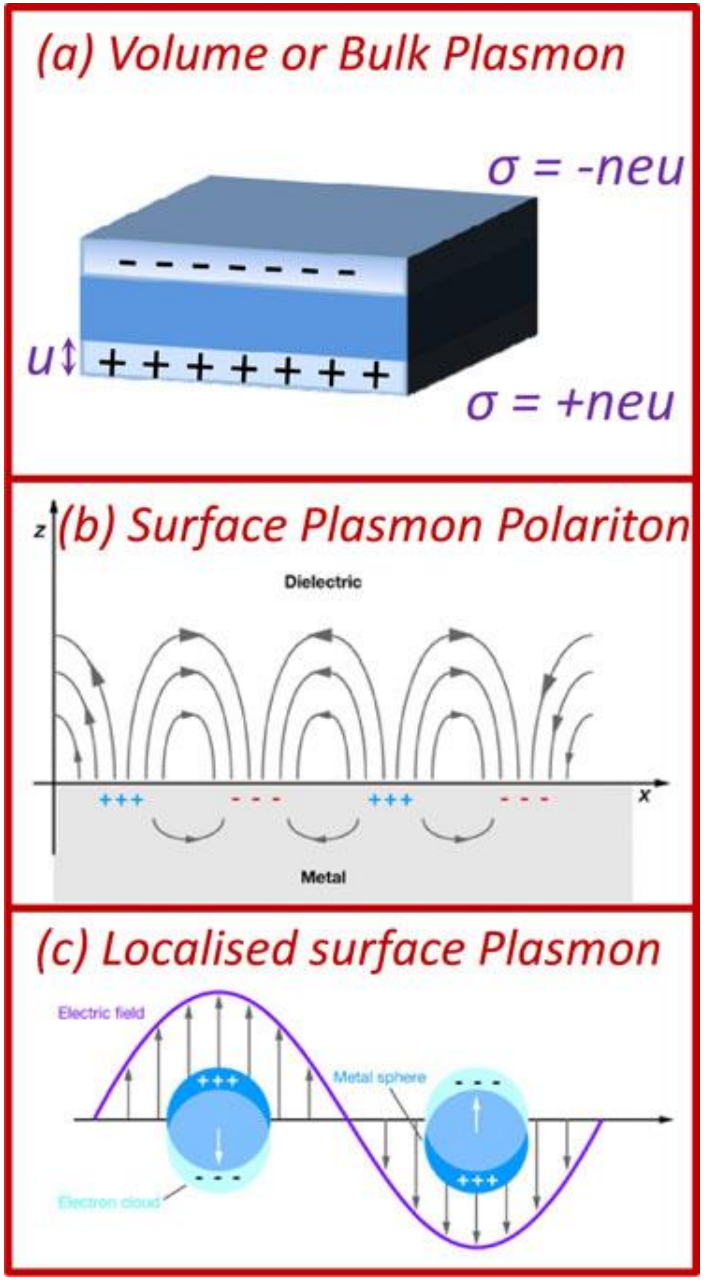
$P_a(\mathbf{R}_e, \omega)$ → Purcell factor for an emitter at $\mathbf{R}_e$, with transition dipole moment oriented along $\hat{\mathbf{e}}_a(\mathbf{R}_e, \omega)$, and frequency ω.

- Once we know the **one-photon** SE rate of an emitter we can obtain immediately the **TPSE spectral density**!

TPSE near plasmonic nanostructures

PLASMONICS

- What is a plasmon?



PLASMONICS

- What is plasmonics?

"You just have Maxwell's equations, some material properties and some boundary conditions, all classical stuff - what's new about that?"

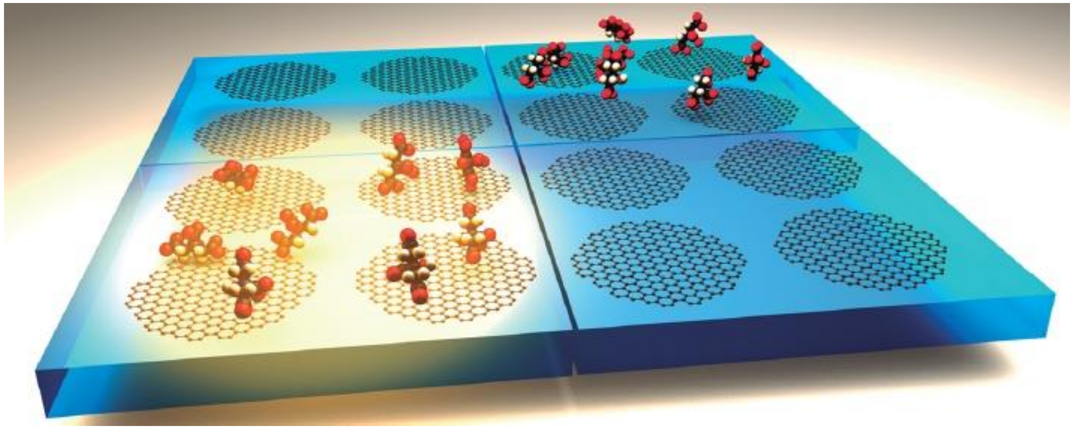
S. A. Maier, *Plasmonics: fundamentals and applications*.

Physics!

- Strong light confinement → beyond the diffraction limit.
- Extreme enhancement of the electromagnetic field intensity → surface physics and **non-linear optics**.

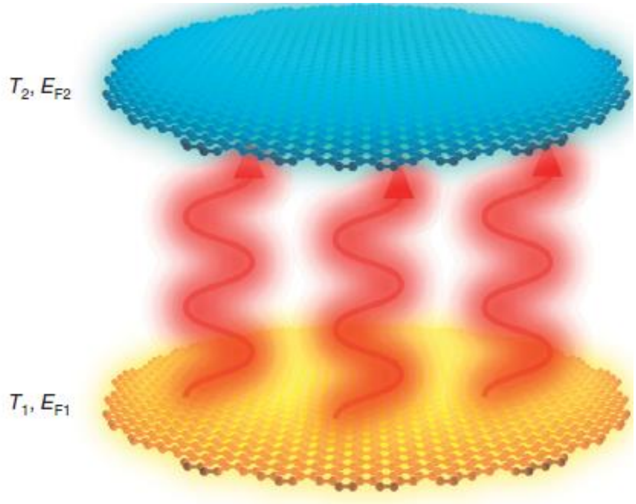
Plasmonics in 2D systems - graphene

- Spatially resolved optical sensing in the infrared



•ACS Photonics 2017, 4, 1831−1838
•*ACS Photonics* 2018, 5, 8, 3282-3290

- Ultrafast radiative heat transfer

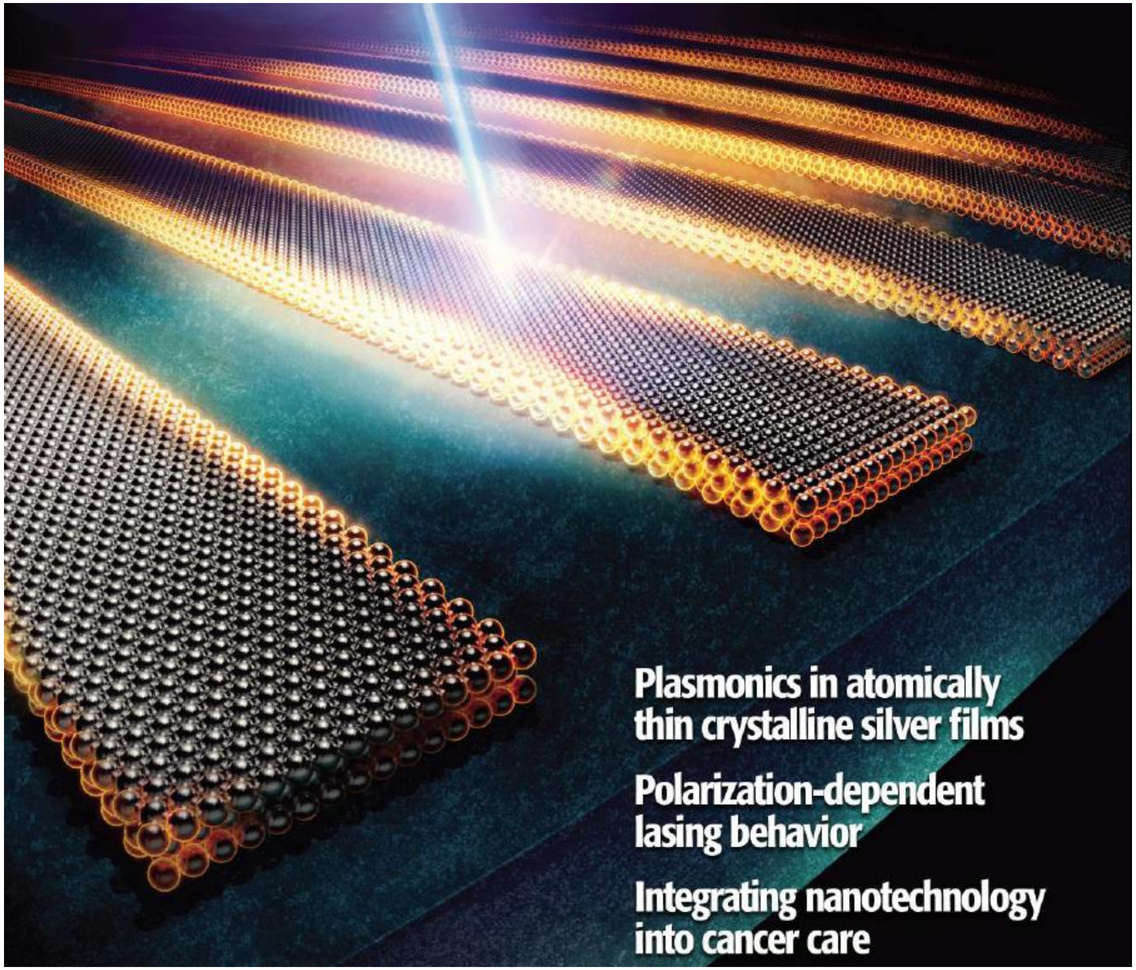


Nature Communications, **8**, 2 (2017)

(Quasi-)2D noble metals

- Wide range of frequencies (visible and near-infrared)

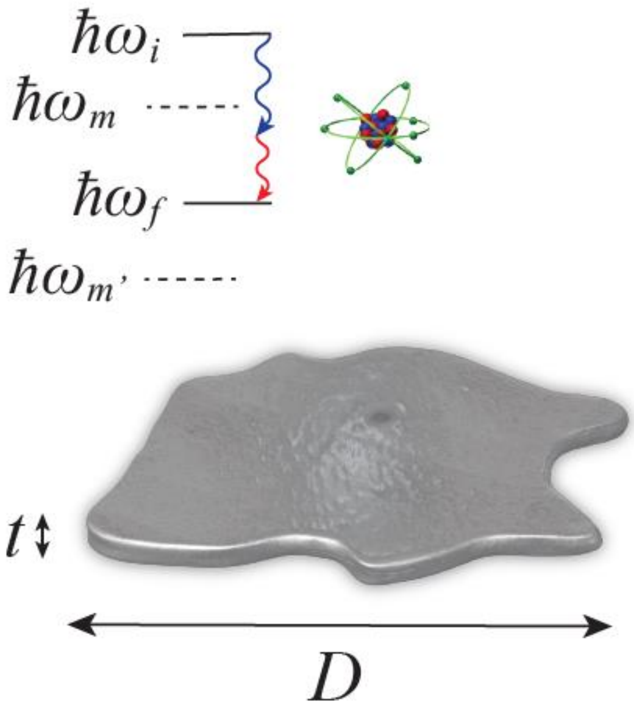
- Recent fabrication of quasi-2D metal films.



•ACS NANO,**13**, 7 (2019)
•Nature Photonics, **8**, 328-333 (2019)

System under study

- An emitter near a 2D plasmonic nanostructure of arbitrary geometry.



Plasmons in 2D nanostructures

- Plasmon Wave Function (**PWF**) formalism:

$$\rho_{2D}(\mathbf{r},\omega) = \frac{4\pi\epsilon_0}{D}\sum_j \frac{c_j}{1/\eta_j - 1/\eta(\omega)} v_j(\mathbf{u})$$ → *Plasmon Wave Function j*

- Resonance frequencies:

$$\mathrm{Re}[1/\eta_j - 1/\eta(\omega_j)] = 0 \qquad \eta(\omega) = i\sigma(\omega)/4\pi\epsilon_0\omega D$$

•ACS Photonics 2017, 4, 3106−3114
•*Faraday Discussions* 2015, 178, 87-107

Plasmons in 2D nanostructures

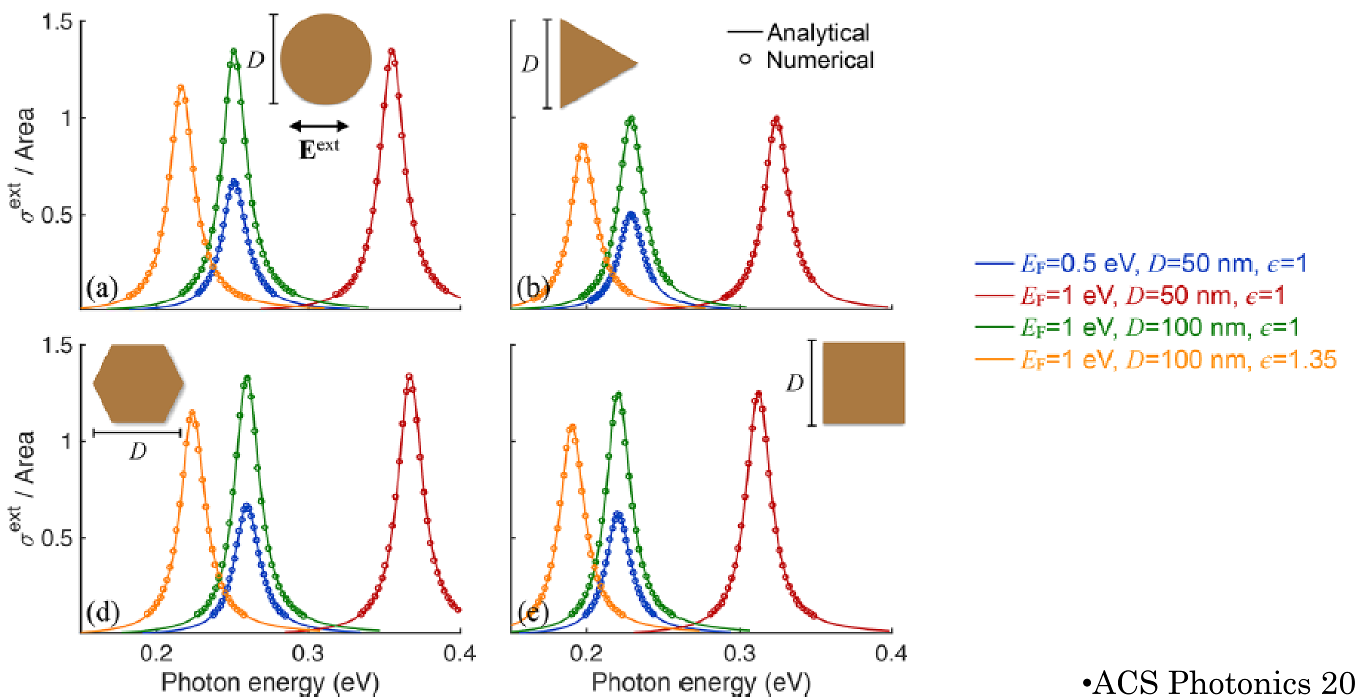
- Plasmon Wave Function (**PWF**) formalism:

$$\rho_{2D}(\mathbf{r},\omega) = \frac{4\pi\epsilon_0}{D}\sum_j \frac{c_j}{1/\eta_j - 1/\eta(\omega)} v_j(\mathbf{u}) \longrightarrow \textit{Plasmon Wave Function } j$$

- Resonance frequencies:

$$\mathrm{Re}[1/\eta_j - 1/\eta(\omega_j)] = 0 \qquad \eta(\omega) = i\sigma(\omega)/4\pi\epsilon_0\omega D$$

- Excelent agreement with numerical calculations.



- ACS Photonics 2017, 4, 3106−3114
- *Faraday Discussions* 2015, 178, 87-107

Purcell factors

- The Purcell factors are numerically equal to the ratio between the power dissipated by an electric dipole near the nanostructure with respect to its free-space radiation rate.

$$P_a(\mathbf{R}_e, \omega) = W_a(\mathbf{R}_e, \omega)/W_0(\omega)$$

Purcell factors

- The Purcell factors are numerically equal to the ratio between the power dissipated by an electric dipole near the nanostructure with respect to its free-space radiation rate.

$$P_a(\mathbf{R}_e,\omega) \;=\; W_a(\mathbf{R}_e,\omega)/W_0(\omega)$$

$$P_a(\mathbf{R}_e,\omega) = P_{a,nr}(\mathbf{R}_e,\omega) + P_{a,r}(\mathbf{R}_e,\omega)$$

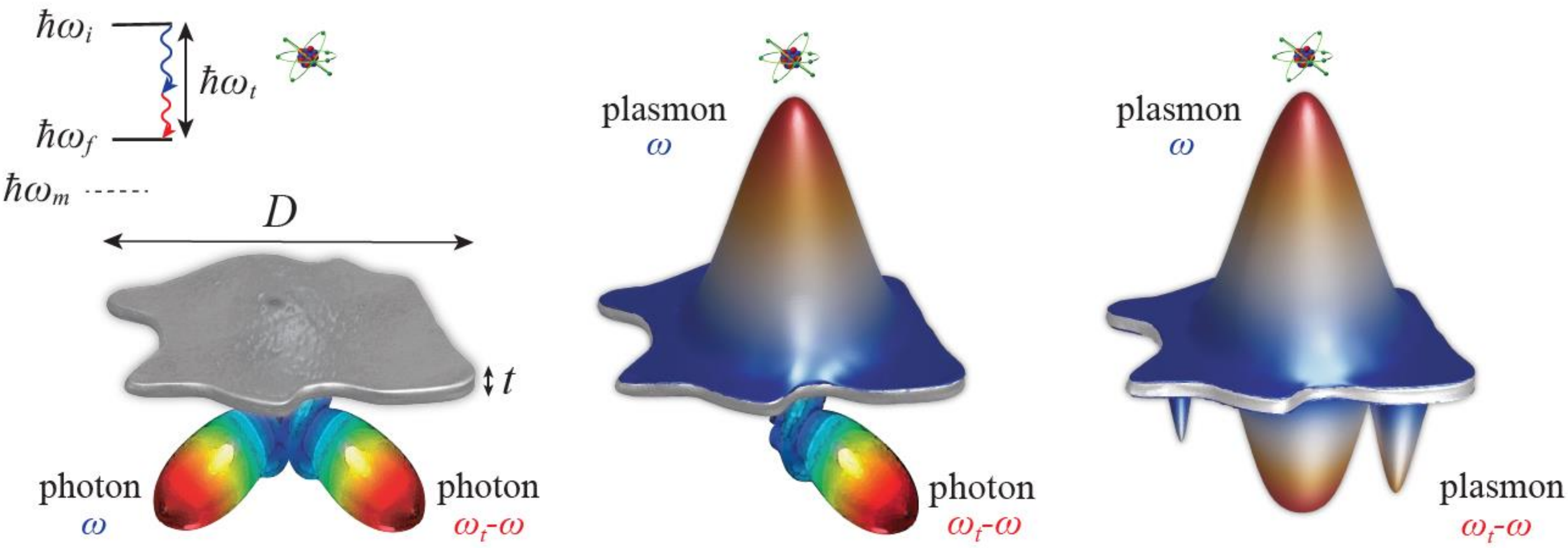
- Absorption (**plasmon emission**):

$$P_{a,nr}(\mathbf{R}_e,\omega) = \frac{6\pi\epsilon_0 c^3}{\omega^4|\mathbf{d}_a|^2}\int d^3\mathbf{R}'\mathrm{Re}\{\mathbf{J}^*(\mathbf{R}',\omega)\cdot\mathbf{E}(\mathbf{R}',\omega)\}$$

- Far-field radiation (**photon emission**):

$$P_{a,r}(\mathbf{R}_e,\omega) = \frac{6\pi\epsilon_0 c^3}{\omega^4|\mathbf{d}_a|^2}\int_{R'\to\infty} d\mathbf{A}'\cdot\mathrm{Re}\{\mathbf{E}(\mathbf{R}',\omega)\times\mathbf{H}^*(\mathbf{R}',\omega)\}$$

TPSE DECAY CHANNELS



$$\gamma(\mathbf{R}_e,\omega) = \gamma_0(\omega)\sum_{a,b} t_{ab}(\omega)P_a(\mathbf{R}_e,\omega)P_b(\mathbf{R}_e,\omega_t-\omega)$$

$$P_a(\mathbf{R}_e,\omega) = P_{a,nr}(\mathbf{R}_e,\omega) + P_{a,r}(\mathbf{R}_e,\omega)$$

- Photon-photon, photon-plasmon and plasmon-plasmon states.

Approximated Purcell factors: Drude model

$$P_{a,nr}(\mathbf{R}_e,\omega) \simeq \sum_{q=1}^{N} \frac{A_{a,q}}{\omega^2} \frac{1/2\tau}{(\omega-\omega_q)^2+(1/2\tau)^2}$$

Plasmonic contribution → **Lorentzian resonances**

Approximated Purcell factors: Drude model

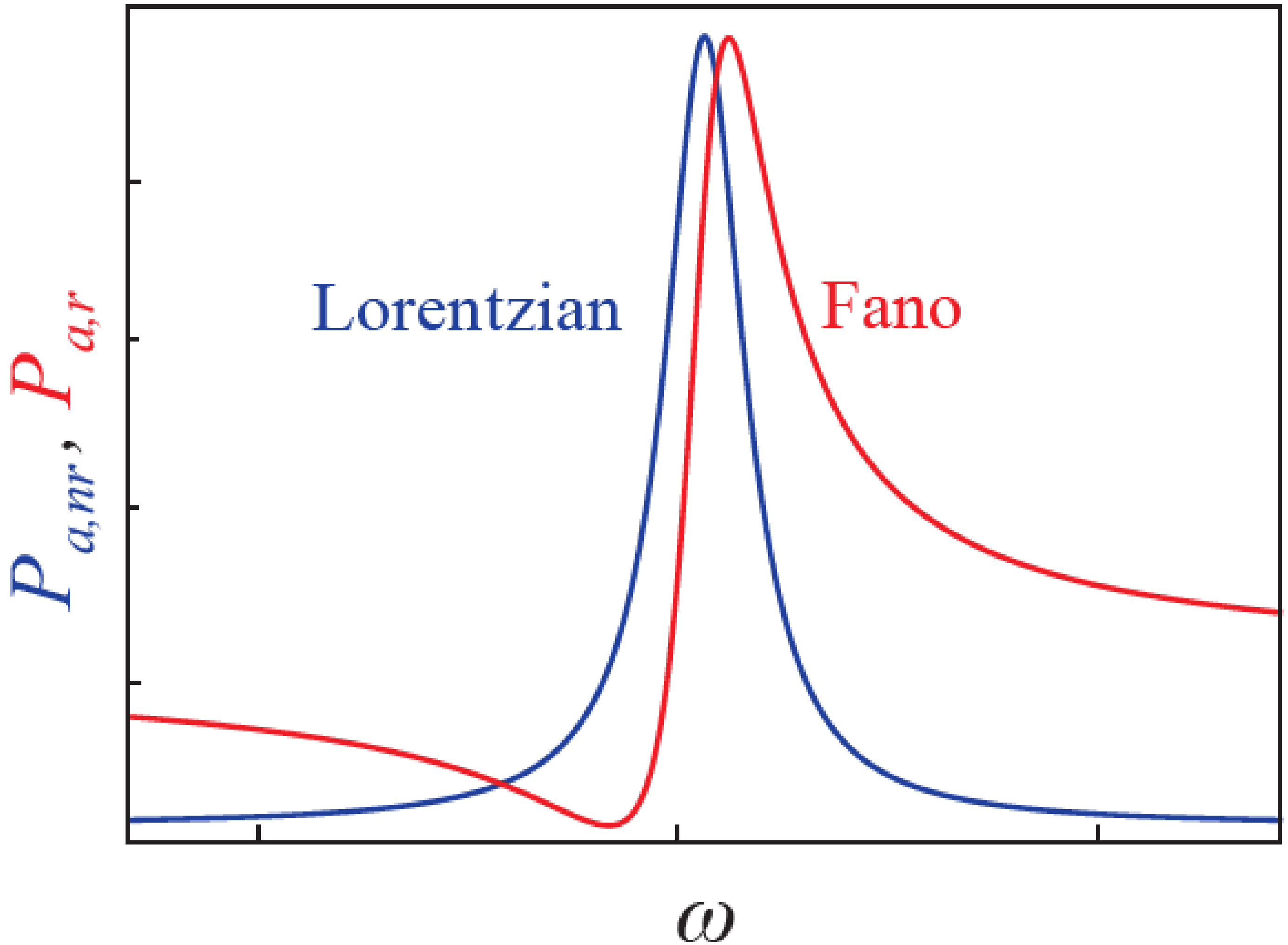
$$P_{a,nr}(\mathbf{R}_e,\omega) \simeq \sum_{q=1}^{N} \frac{A_{a,q}}{\omega^2} \frac{1/2\tau}{(\omega-\omega_q)^2+(1/2\tau)^2}$$

Plasmonic contribution → **Lorentzian resonances**

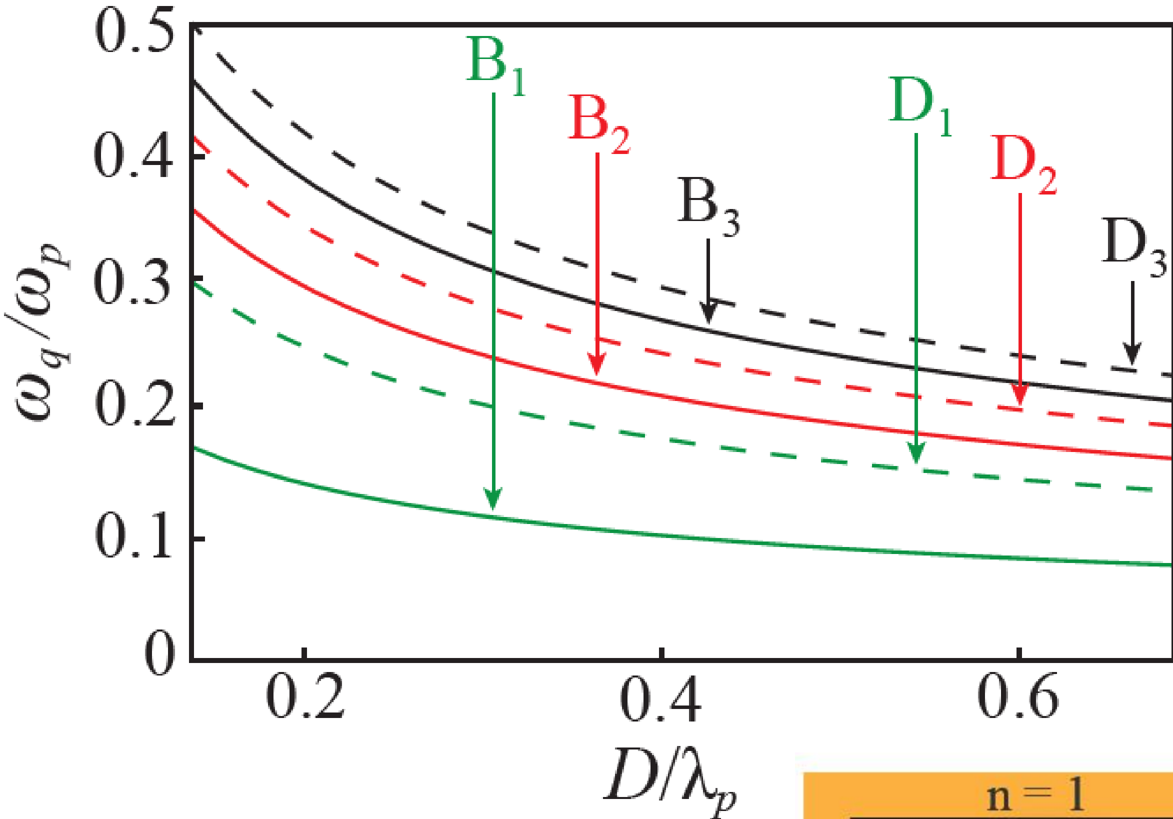
$$P_{a,r}(\omega) \simeq \sum_{q=1}^{N} \frac{B_{a,q}(1/2\tau)^2+(\omega-\omega_q+f_{a,q}/2\tau)^2}{(\omega-\omega_q)^2+(1/2\tau)^2} - (N-1)$$

Photonic contribution → **Fano + Lorentzian resonances**

Approximated results: Drude model



A METALLIC NANODISK CLOSE TO AN ON-AXIS QUANTUM EMITTER: RESONANT MODES

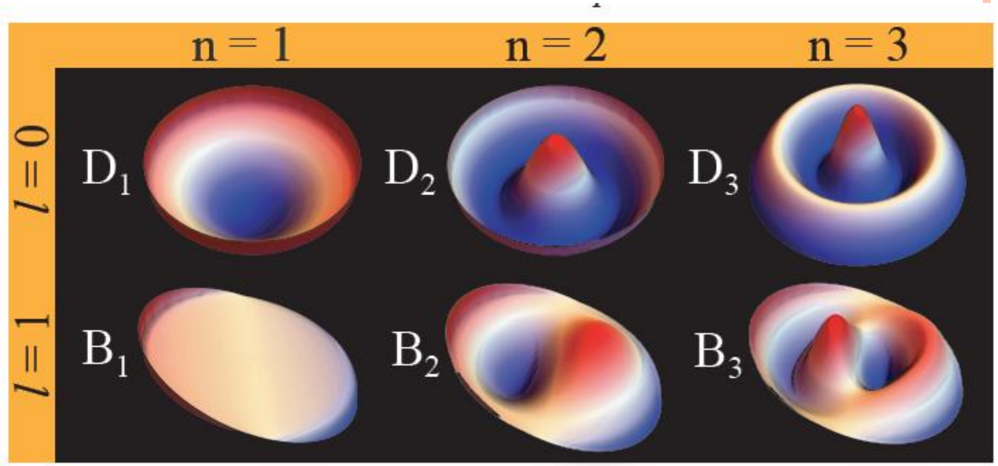


Bilayer of Ag(111)

$\hbar\omega_p = 2\pi\hbar c/\lambda_p = 9.1$ eV

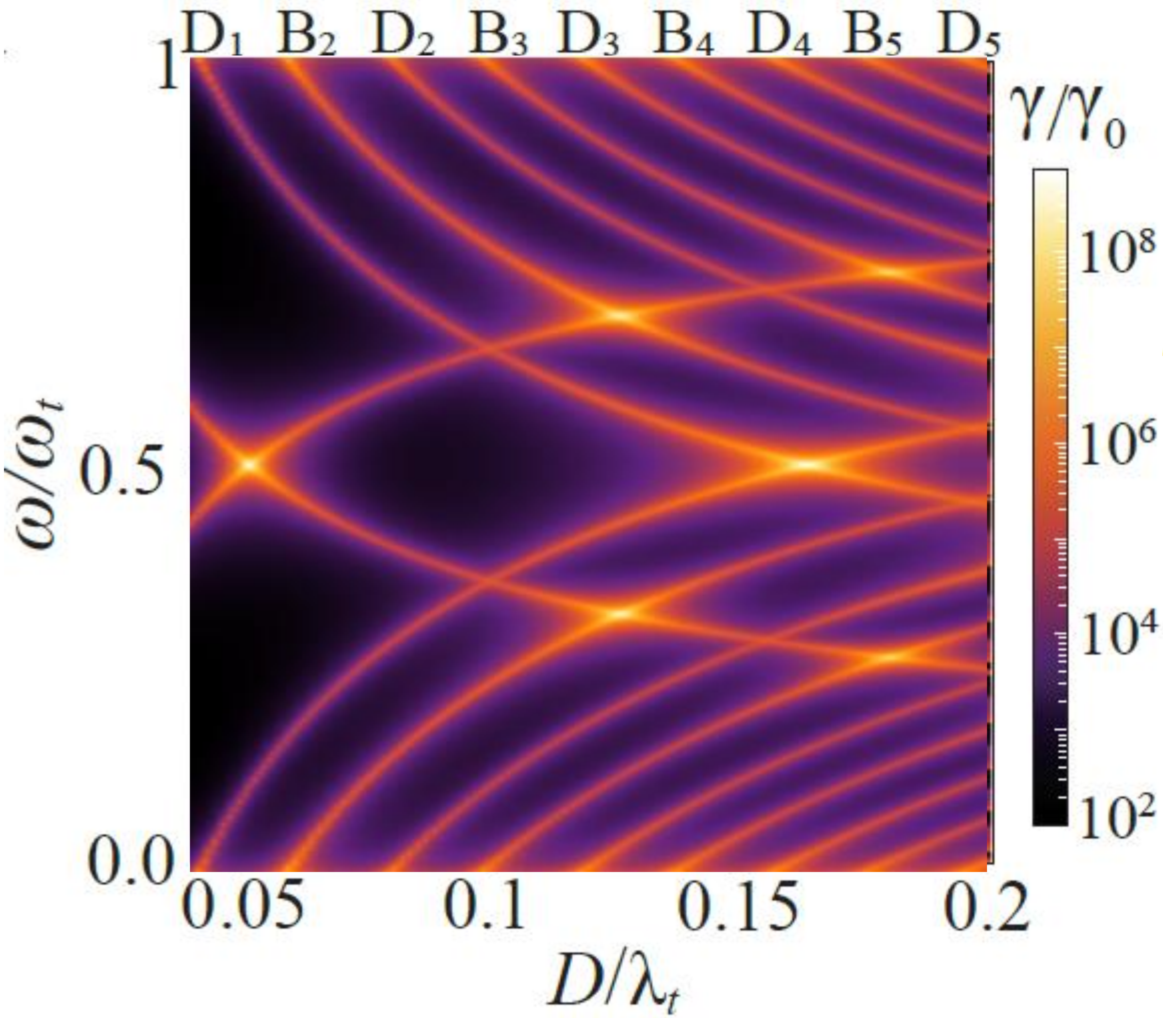
$\hbar\tau^{-1} = 18$ meV

- **Resonance frequencies** can be **tuned** by changing the **size** of the nanodisk.



A METALLIC NANODISK CLOSE TO AN ON-AXIS QUANTUM EMITTER: TPSE SPECTRUM

- **Crossings** between **Bright-Bright** or **Dark-Dark** modes produce **extreme enhancements**.



$\hbar\omega_t \simeq 2.64\,\text{eV}$ ($\lambda_t \simeq 470$ nm)

$z_e = 10$ nm

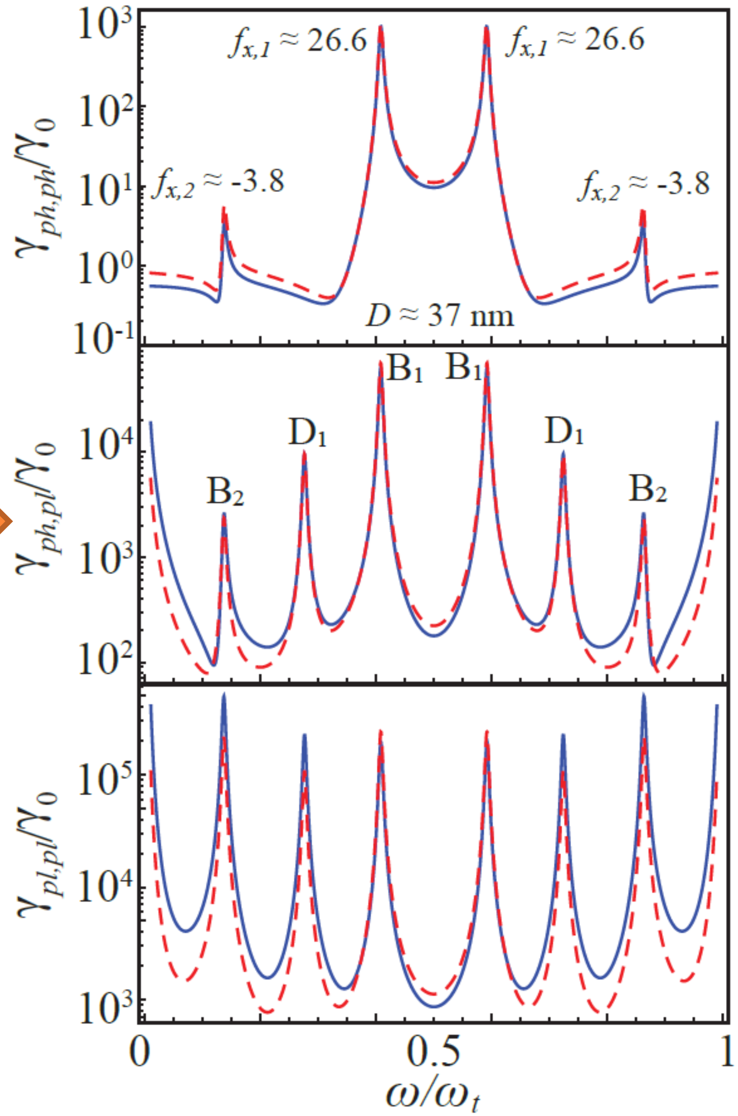
A metallic nanodisk close to an on-axis quantum emitter: decay channels

- Spectral line-shapes serve as **fingerprints** of the three **decay channels**.

Photon-photon channel (Fano-Fano)

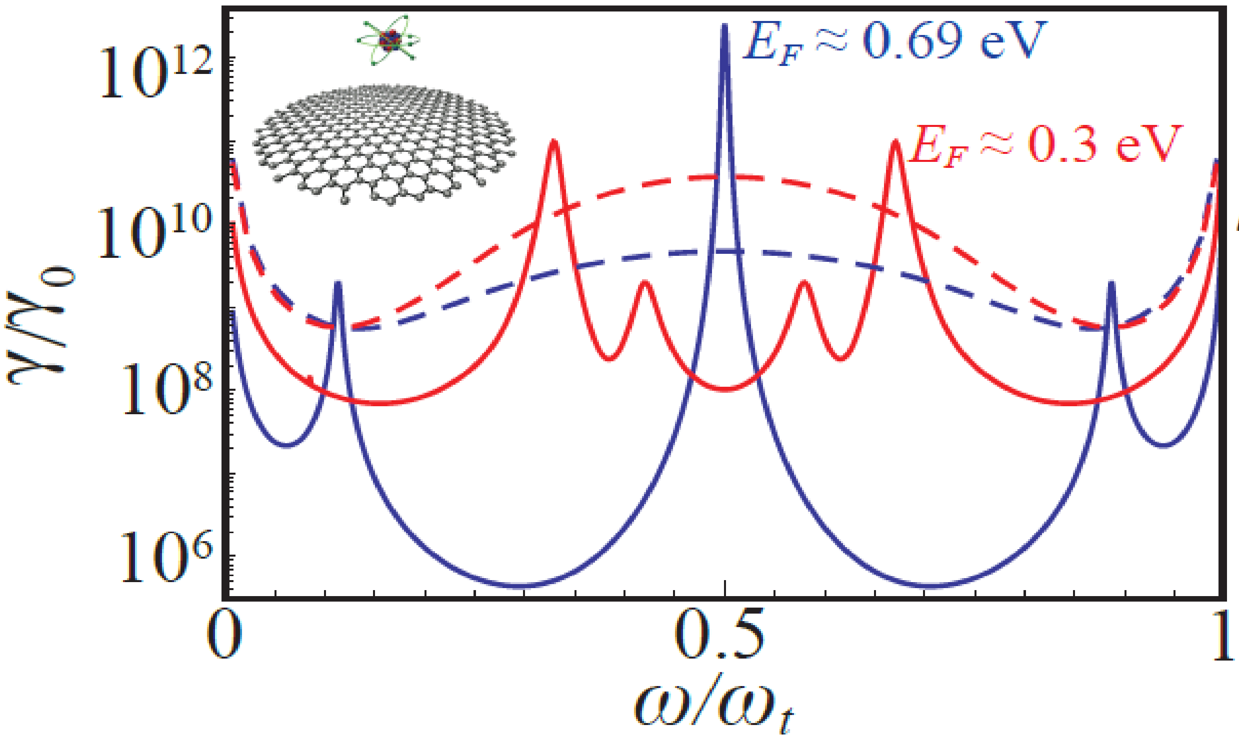
Photon-plasmon channel (Fano-Lorentzian)

Plasmon-plasmon channel (Lorentzian-Lorentzian)



Dynamical control of TPSE with a graphene nanodisk

- **Enhanced selective spectral emission** compared to the typical broadband spectrum of monolayers.



$D = 40$ nm

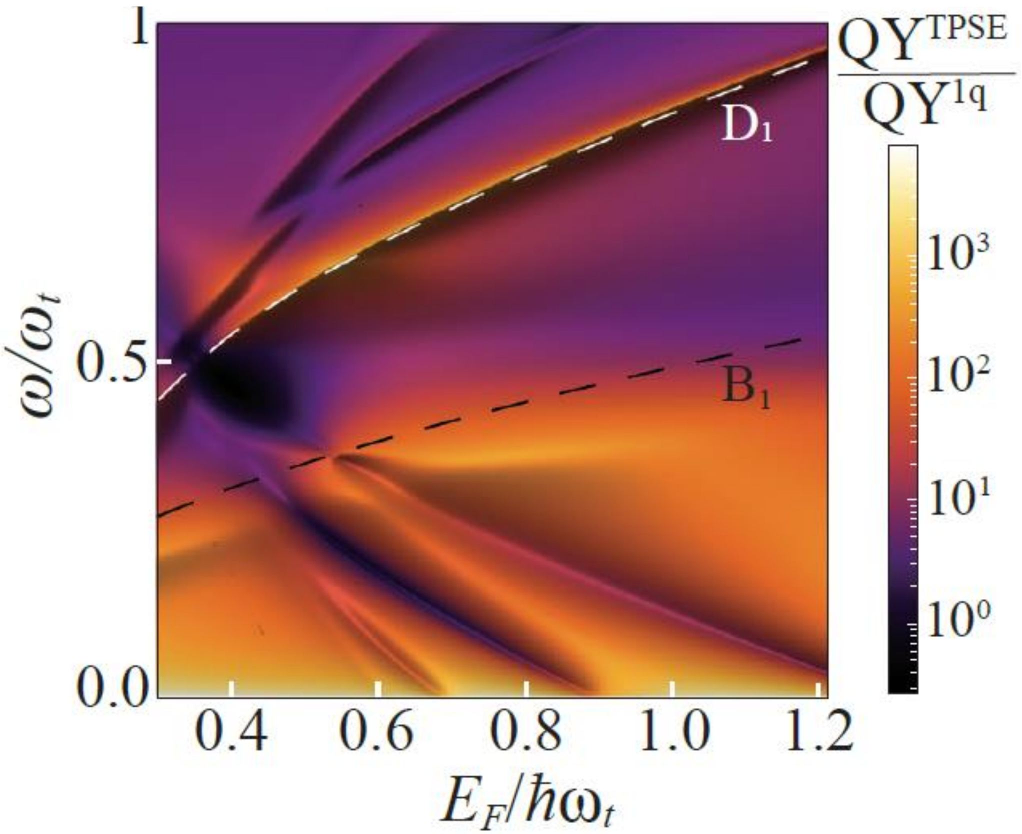
$\mu = 2500$ cm^2 V^{-1} s^{-1}

$\hbar\omega_t \simeq 0.66$ eV

$z_e = 10$ nm

DYNAMICAL CONTROL OF TPSE WITH A GRAPHENE NANODISK

$$\mathrm{QY}^{1q}(\omega) = \frac{\gamma^{1q}_{ph}(\omega)}{\gamma^{1q}(\omega)}, \quad \mathrm{QY}^{\mathrm{TPSE}}(\omega) = \frac{\gamma_{ph,ph}(\omega) + \gamma_{ph,pl}(\omega)}{\gamma(\omega)}$$



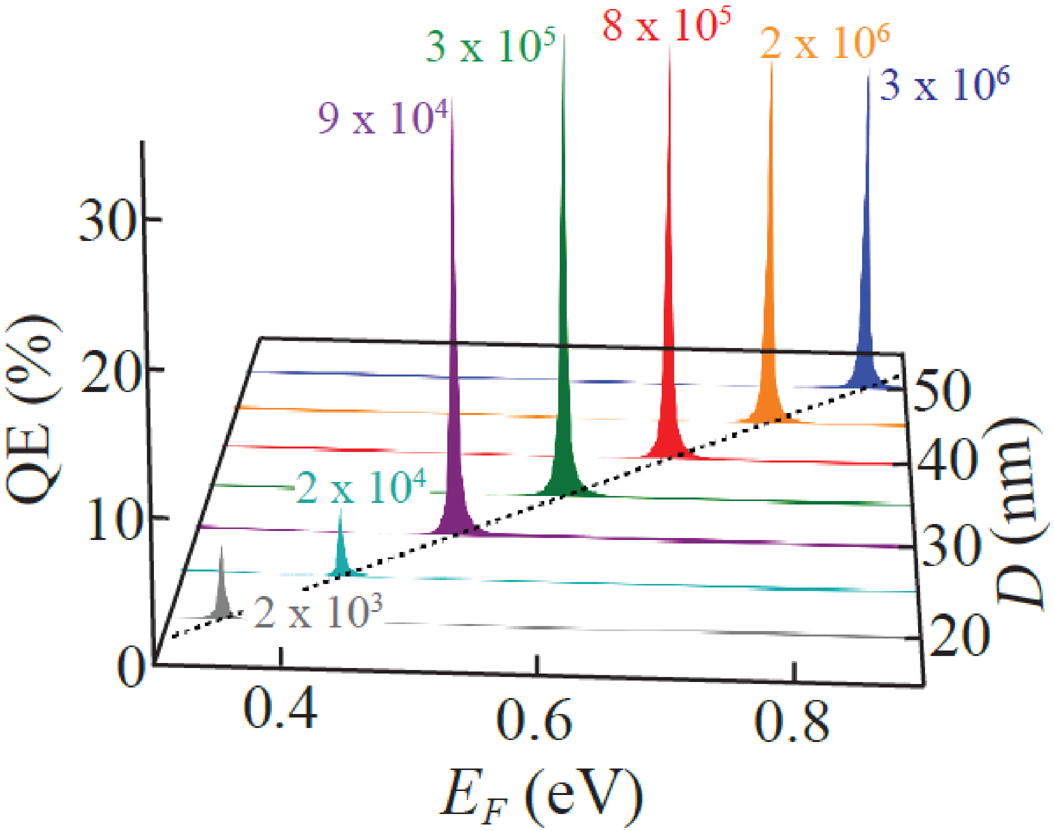
The fundamental dark mode acts as an amplifier of the photon-plasmon channel, but as an attenuator of the one-photon emission pathway.

- Single photon creation via a two-quanta process can be much more efficient than standard one-photon emission.

Dynamical control of TPSE with a graphene nanodisk

$$\mathrm{QE} = \Gamma_{4s\rightarrow 3s}/(\Gamma_{4s\rightarrow 3s} + \gamma^{1q}_{4s\rightarrow 3p} + \gamma^{1q}_{4s\rightarrow 2p})$$

- For any disk diameter the quantum efficiency can be optimized by tuning the Fermi energy so that $\omega_{B_1} = \omega_t/2$.



$\mu = 10^4\ \mathrm{cm^2\ V^{-1}\ s^{-1}}$

Numbers near curves:
ph-ph Purcell factors

Conclusions

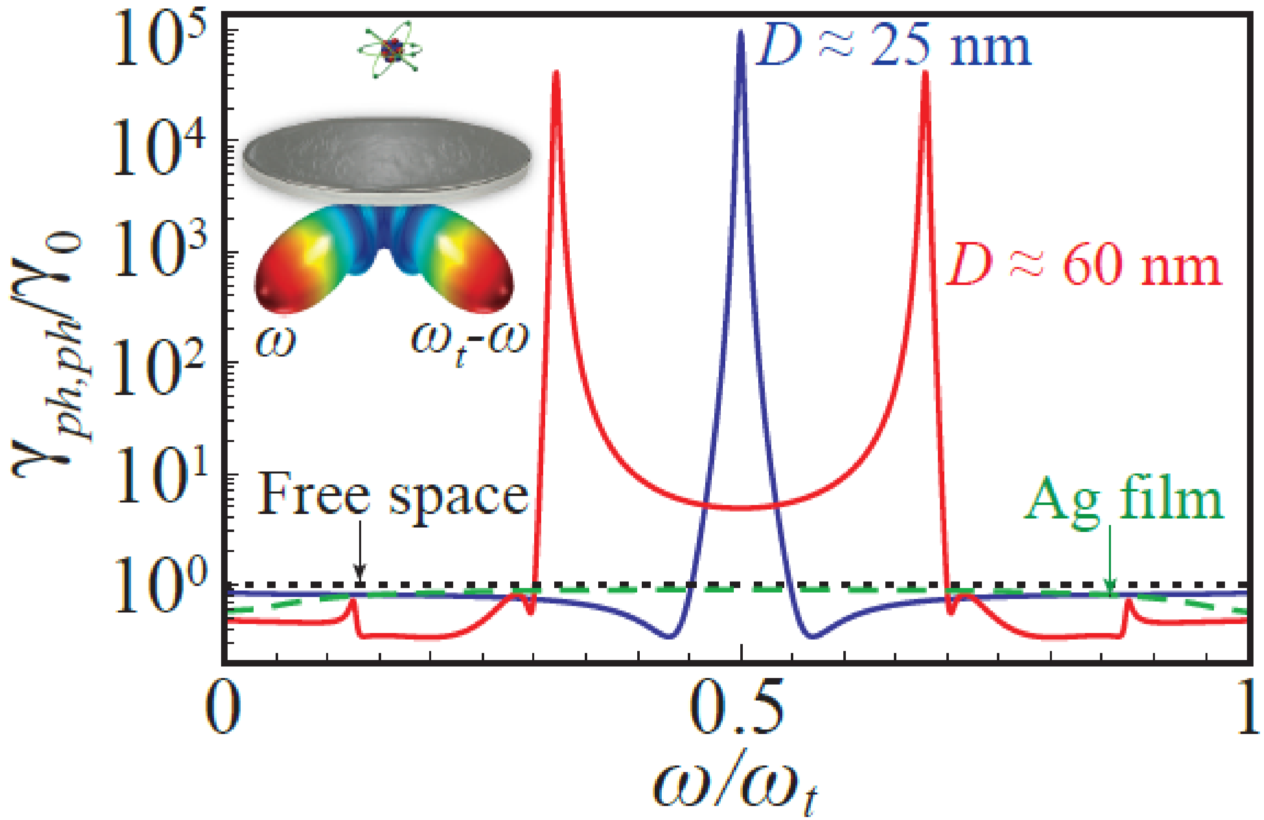
- It is possible to pre-select the frequencies of emission by tuning the size and doping of the nanostructure.
- 2D plasmonic nanostructures allow enhanced TPSE rate with **generation of photons**, not only plasmons.
- Surprisingly, **TPSE** can be a **single photon source** orders of magnitude more efficient than one-photon SE.
- **Finite-sized** plasmonic systems have many **advantages** over **extended** ones.

THANK YOU!

Additional information

A metallic nanodisk close to an on-axis quantum emitter: Ph-ph decay channel

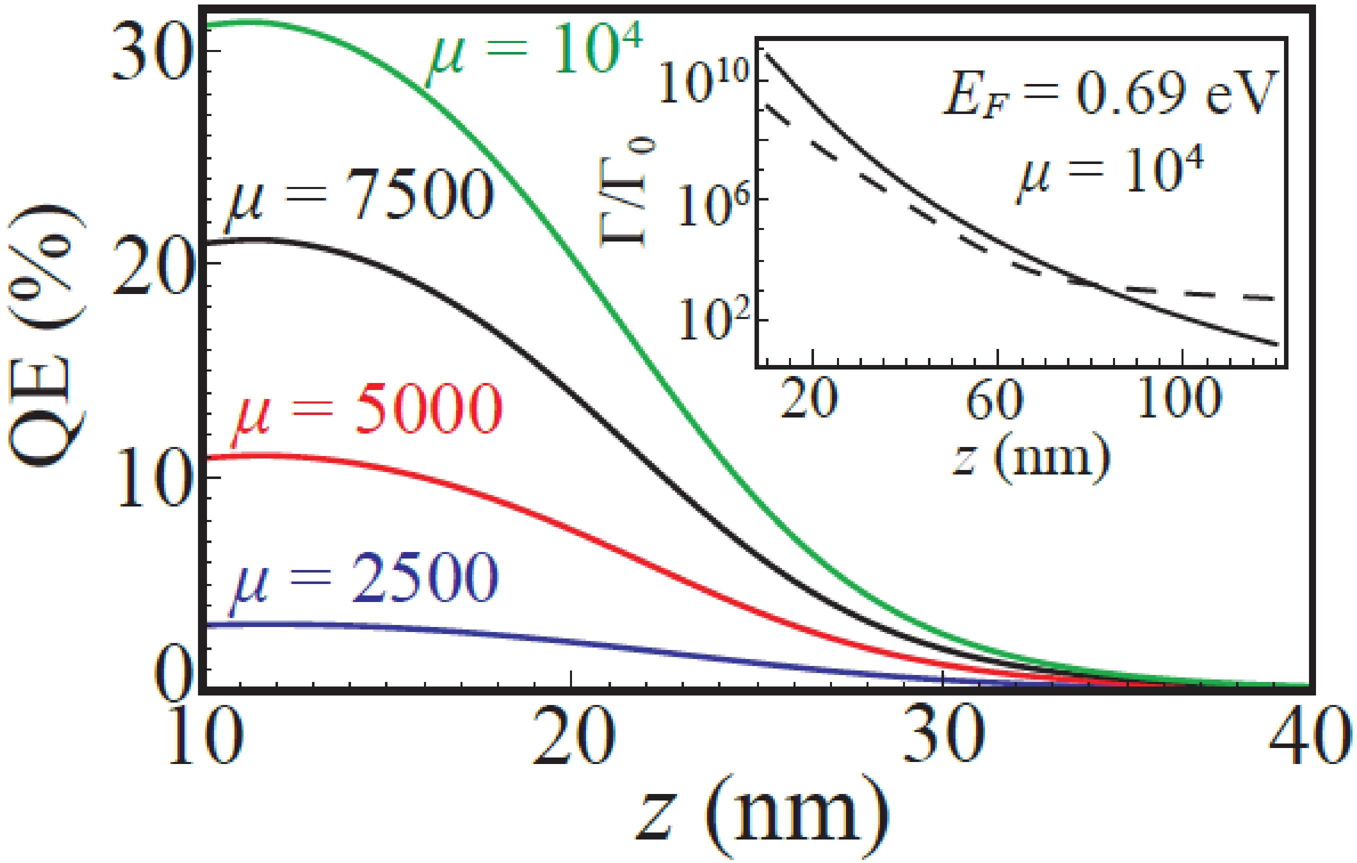
- **Finite size** is critical to accomplish **giant photon-photon production**.



Dynamical control of TPSE with a graphene nanodisk

- Graphene nanostructures disrupt the usual unbalance between one- and two-quanta emission.

$$\mathrm{QE} = \Gamma_{4s\to 3s}/(\Gamma_{4s\to 3s} + \gamma^{1q}_{4s\to 3p} + \gamma^{1q}_{4s\to 2p})$$



Purcell effect

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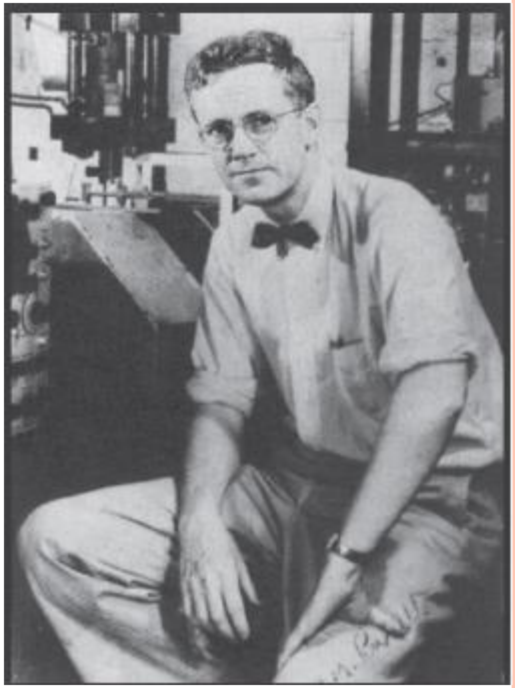

- Reason: the presence of the bodies affects the **boundary conditions** (BC) on the electromagnetic field.

$$\Gamma(\mathbf{R}) = \frac{\pi}{\epsilon_o \hbar} \sum_{\mathbf{k}p} \omega_k \left| \mathbf{d}_{eg} \cdot \mathbf{A}_{\mathbf{k}p}(\mathbf{R}) \right|^2 \delta(\omega_k - \omega_{eg}).$$

$$\frac{\Gamma}{\Gamma_o} = \frac{6\pi c}{\omega_{eg}} \hat{\mathbf{n}}_{eg}^* \cdot \left[\mathrm{Im}\mathbb{G}(\mathbf{R}, \mathbf{R}, \omega_{eg}) \right] \cdot \hat{\mathbf{n}}_{eg},$$

$$\nabla \times \nabla \times \mathbb{G}(\mathbf{r}, \mathbf{r}', \omega) - \frac{\omega^2}{c^2} \mathbb{G}(\mathbf{r}, \mathbf{r}', \omega) = \mathbb{I}\delta(\mathbf{r} - \mathbf{r}').$$

L. Novotny and **B. Hecht**, *Principles of nano-optics*. Cambridge university press, 2012.

Green's function method

- The imaginary part of the Green's function can be written in terms of the field modes as

$$\mathrm{Im}\mathbb{G}(\mathbf{r},\mathbf{r}',\omega) = \frac{\pi c^2}{2\omega}\sum_{\mathbf{k}p}\mathbf{A}^*_{\mathbf{k}p}(\mathbf{r}')\mathbf{A}_{\mathbf{k}p}(\mathbf{r})\delta(\omega-\omega_k).$$

- Using the previous identity, we recover the well known expression for the TPSE rate, namely

$$\Gamma = \frac{\mu_0^2}{\pi\hbar^2}\int_0^{\omega_{eg}} d\omega\,\omega^2(\omega_{eg}-\omega)^2\mathrm{Im}\mathbb{G}_{il}(\omega)\mathrm{Im}\mathbb{G}_{jn}(\omega_{eg}-\omega)\mathbb{D}_{ij}(\omega,\omega_{eg}-\omega)\mathbb{D}^*_{ln}(\omega,\omega_{eg}-\omega).$$

N. Rivera et al., Science, vol. **353**, no. **6296**, pp. **263–269** (**2016**).

- This constitutes an **alternative demonstration** of this formula!

The Purcell factors relation

- Choosing the basis which diagonalizes the Green's function we have

$$\gamma(\omega) = \frac{\mu_0^2}{\pi\hbar^2}\omega^2(\omega_{eg}-\omega)^2\sum_{i,j}\mathrm{Im}\mathbb{G}_{ii}(\omega)\mathrm{Im}\mathbb{G}_{jj}(\omega_{eg}-\omega)|\mathbb{D}_{ij}(\omega,\omega_{eg}-\omega)|^2.$$

- We define the **Purcell factors** as

$$P_i(\mathbf{R},\omega) := \frac{6\pi c}{\omega}\mathrm{Im}\mathbb{G}_{ii}(\mathbf{R},\mathbf{R},\omega).$$

- In this way, we can write

$$\frac{\gamma(\omega)}{\gamma_o(\omega)} = \sum_{i,j}\frac{|\mathbb{D}_{ij}(\omega,\omega_{eg}-\omega)|^2}{|\mathbb{D}(\omega,\omega_{eg}-\omega)|^2}P_i(\omega)P_j(\omega_{eg}-\omega).$$

- The **TPSE** rate **dependence** on the **local density of states** (LDOS) was made explicit!

Plasmons in 2D nanoestructures

- Plasmon Wave Function (**PWF**) formalism:

$$\rho_{2D}(\mathbf{r},\omega) = \frac{4\pi\epsilon_0}{D}\sum_j \frac{c_j}{1/\eta_j - 1/\eta(\omega)} v_j(\mathbf{u}),$$

$$v_j(\mathbf{u}) = \nabla_{\mathbf{u}} \cdot \sqrt{f(\mathbf{u})}\mathbf{V}_j(\mathbf{u})$$ → *Plasmon Wave Functions*

$$\int d^2\mathbf{u}'\, \mathbb{M}(\mathbf{u},\mathbf{u}') \cdot \mathbf{V}_j(\mathbf{u}') = \frac{1}{\eta_j}\mathbf{V}_j(\mathbf{u}) \,.$$

$$\mathbb{M}(\mathbf{u},\mathbf{u}') = \sqrt{f(\mathbf{u})f(\mathbf{u}')}\nabla_{\mathbf{u}}\nabla_{\mathbf{u}'}|\mathbf{u}-\mathbf{u}'|^{-1}$$

- Resonance frequencies:

$$\mathrm{Re}[1/\eta_j - 1/\eta(\omega_j)] = 0 \qquad \eta(\omega) = i\sigma(\omega)/4\pi\epsilon_0\omega D$$

- External field dependence:

$$c_j = \int d^2\mathbf{u}\, \mathbf{V}_j^*(\mathbf{u}) \cdot \boldsymbol{\mathcal{E}}^{ext}(\mathbf{u},\omega) \,.$$

•ACS Photonics 2017, 4, 3106−3114
•*Faraday Discussions* 2015, 178, 87-107

Power dissipated by absorption

$$\mathbf{J}(\mathbf{R}',\omega) = \mathbf{K}(\mathbf{r}',\omega)\delta(z') = \sigma(\omega)f(\mathbf{r}')\mathbf{E}_{\parallel}(\mathbf{r}',\omega)\delta(z')$$

+

$$\boldsymbol{\mathcal{E}}(\mathbf{u},\omega) = \sum_\alpha \frac{c_\alpha}{1-\eta(\omega)/\eta_\alpha}\mathbf{V}_\alpha(\mathbf{u}),\ c_\alpha = \int d^2\mathbf{u}\,\mathbf{V}^*_\alpha(\mathbf{u})\cdot\boldsymbol{\mathcal{E}}^{ext}(\mathbf{u},\omega)$$

$$\mathbf{E}^{ext}(\mathbf{R}',\omega) = \frac{1}{4\pi\epsilon_0}\nabla\mathbf{d}_a\cdot\nabla|\mathbf{R}-\mathbf{R}'|^{-1}$$

+

$$\int d^2\mathbf{u}\,\mathbf{V}^*_\alpha(\mathbf{u})\cdot\mathbf{V}_{\alpha'}(\mathbf{u}) = \delta_{\alpha\alpha'}.$$

$$P_{a,nr}(\mathbf{R}_e,\omega) = \frac{3c^3}{2D^3\omega^3}\mathrm{Im}\sum_\alpha \hat{\mathbf{e}}_a\cdot\frac{\mathbf{F}_\alpha(\mathbf{R}_e)\otimes\mathbf{F}^*_\alpha(\mathbf{R}_e)}{1/\eta(\omega)-1/\eta_\alpha}\cdot\hat{\mathbf{e}}_a.$$

$$\mathbf{F}_\alpha(\mathbf{R}_e) = \int d^2\mathbf{u}'\frac{v_\alpha(\mathbf{u}')(\mathbf{R}_e/D-\mathbf{u}')}{|\mathbf{R}_e/D-\mathbf{u}'|^3}$$

Power dissipated by radiation

- The system is spatially localized, therefore we can make a multipole expansion. The first contribution to the power radiated by the system is

$$P_{a,r}(\mathbf{R}_e,\omega) \simeq \frac{|\mathbf{d}_a + \mathbf{d}_{a,ind}(\mathbf{R}_e,\omega)|^2}{|\mathbf{d}_a|^2}$$

$$\mathbf{d}_{a,ind}(\mathbf{R}_e,\omega) = \int d^2\mathbf{r}\,\mathbf{r}\,\rho_{2D}(\mathbf{r},\omega)$$

$$\rho_{2D}(\mathbf{r},\omega) = \frac{4\pi\epsilon_0}{D}\sum_\alpha \frac{c_\alpha}{1/\eta_\alpha - 1/\eta(\omega)} v_\alpha(\mathbf{u})$$

$$P_{a,r}(\mathbf{R}_e,\omega) = \left|\hat{\mathbf{e}}_a + \sum_\alpha \frac{\zeta_\alpha \otimes \mathbf{F}^*_\alpha(\mathbf{R}_e)}{1/\eta_\alpha - 1/\eta(\omega)} \cdot \hat{\mathbf{e}}_a\right|^2. \quad \zeta_\alpha = \int d^2\mathbf{u}\,\mathbf{u}\,v_\alpha(\mathbf{u})$$

Approximated results: Drude model

$$P_{a,nr}(\mathbf{R}_e,\omega) \simeq \sum_{q=1}^{N} \frac{A_{a,q}}{\omega^2} \frac{1/2\tau}{(\omega-\omega_q)^2+(1/2\tau)^2}$$

$$A_{a,q} = \frac{3c^3\omega_p^2 t}{16\pi D^4 \omega_q^2} \sum_{j=1}^{g_q} |\hat{\mathbf{e}}_a \cdot \mathbf{F}_{q,j}(\mathbf{R}_e)|^2 \,.$$

$$P_{a,r}(\omega) \simeq \sum_{q=1}^{N} \frac{B_{a,q}(1/2\tau)^2+(\omega-\omega_q+f_{a,q}/2\tau)^2}{(\omega-\omega_q)^2+(1/2\tau)^2} - (N-1)$$

$$f_{a,q} = \frac{\omega_p^2 \tau t}{4\pi D\omega_q} \sum_{j=1}^{g_q} \mathrm{Re}\left[\hat{\mathbf{e}}_a \cdot \mathbf{F}^*_{q,j}(\mathbf{R}_e)\zeta^{\|}_{a;q,j}\right]$$

Plasmons in 2D nanostructures

- How do we obtain the PWFs?

$$\int d^2\mathbf{u}'\,\mathbb{M}(\mathbf{u},\mathbf{u}')\cdot\mathbf{V}_j(\mathbf{u}') = \frac{1}{\eta_j}\mathbf{V}_j(\mathbf{u})\,.$$

Homogeneous medium!

$$\nabla^2_{\mathbf{u}}\int d^2\mathbf{u}'\,\frac{v_j(\mathbf{u}')}{|\mathbf{u}-\mathbf{u}'|} = \frac{1}{\eta_j}v_j(\mathbf{u})\,.$$

Poisson equation over the nanostructure → **Laplace equation + BC over the nanostructure**

- Or numerical methods.

Plasmons in a nanodisk

- Analytical solution!

PWFs = $R_{ln}(u)e^{il\phi}$. $\quad R_{ln}(u) = (2u)^{|l|}\sum_{m'} a^{ln}_{m'} P^{(|l|,0)}_{m'}(1-8u^2)$

$$\mathbb{G}^l \mathbf{a}^{ln} = -4\pi\eta_{ln}\mathbb{K}^l\mathbf{a}^{ln},$$

$$\mathbb{K}^l_{mm'} = \frac{(-1)^{m-m'+1}}{\pi[4(m-m')^2-1](|l|+m+m'+1/2)(|l|+m+m'+3/2)}, \quad m,m' = 0,1,2,3...$$

$$\mathbb{G}^l_{mm'} = \frac{\delta_{m0}\delta_{m'0}}{8|l|(|l|+1)^2} + \frac{\delta_{mm'}}{4(|l|+2m')(|l|+2m'+1)(|l|+2m'+2)} + \frac{\delta_{m+1,m'}}{8(|l|+2m+1)(|l|+2m+2)(|l|+2m+3)}$$
$$+ \frac{\delta_{m,m'+1}}{8(|l|+2m'+1)(|l|+2m'+2)(|l|+2m'+3)}, \quad m,m' = 0,1,2,3...$$

•PRB 1986, **33,** 5221
•PRB 2016, **93,** 035426